DYNAMIC PROPERTIES OF HEAT RECEIVERS WITH MOVING HEAT CARRIER

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Frequency characteristics of heat receivers with a moving heat carrier are found.

Heat receivers with a moving heat carrier (HRMC) are used to measure the power of electromagnetic radiation. They allow us to carry out measurements in a broad frequency range – from zero (constant current) to frequencies of the Roentgen range – and in a broad power interval – from a few tens of microwatts to tens of kilowatts.

When calculating such meters it is important to know the dynamic properties of HRMC, since they determine the response of the receiver to an input signal. This is required when determining the speed of the meter, the calculation of the magnitude of pulsations of the output signal when a periodically varying radiation acts on it.

In [1, 2] dynamic properties of HRMC are described with the assumption that the material of the receiver is characterized by an infinitely large thermal diffusivity. This assumption, in practice, is not fulfilled in the majority of cases. In the case of a more rigorous consideration we have to include space–time processes inside the receiver itself, taking into account the basic geometric and thermophysical parameters which determine the course of the dynamic processes.

The investigation of the dynamic properties of HRMC is carried out on a model constituting a plate of length b, thickness m, and width l, onto one side of which the radiation falls, while the other side is washed by a stream of heat carrier (HC) moving along the edge of length b. We shall assume that: a) the thermal diffusivity of the HC is zero; b) in the section of the plate transverse with respect to the direction of motion of the HC and in the HC temperature gradients are absent; c) energy losses to the surrounding medium are absent. In the case of these assumptions the dynamics of thermal processes in HRMC can be described by means of a system of two differential equations set up for elementary volumes of HC and plates:

$$\text{a)}\ -c\gamma v\frac{\partial T}{\partial x}dxdt=\frac{C_l}{b}\frac{\partial T}{\partial t}dxdt+\alpha l(T-U)dxdt,$$

$$\text{b)}\ kml\frac{\partial^2 U}{\partial x^2}dxdt+\alpha l(T-U)dtdx+Ndxdt=\frac{C_m}{b}\frac{\partial U}{\partial t}dxdt, \qquad (1)$$

where x is the coordinate measured in the longitudinal (along the propagation of the stream of HC) direction; the values x = 0 and x = b correspond to the entry of HC into the receiver and its exit from it;

$$U=U(x,\ t),\ T=T(x,\ t),\ N=N(x,\ t),\ P(t)=\int_0^b N(x,\ t)\,dx.$$

We introduce the notation

$$\beta=\alpha bl/c\gamma v,\ a^2=kmbl/C_m,\ \tau_m=C_m/c\gamma v,\ \tau_l=C_l/c\gamma v.$$

Then the system of equations (1) can be transformed into

$$\text{a)}\ b\frac{\partial T}{\partial x}+\tau_l\frac{\partial T}{\partial t}+\beta(T-U)=0,$$

$$\text{b)}\ \frac{\partial U}{\partial t}=a^2\frac{\partial^2 U}{\partial x^2}+\frac{\beta}{\tau_m}(T-U)-\frac{Nb}{C_m}. \qquad (2)$$

The solution of the problem being considered will be sought for zero initial conditions for the case of heat insulation of the ends of the plate:

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$$T(x,\ 0)=0,\ U(x,\ 0)=0,\ T(0,\ t)=T_0(t)=0,$$

$$T(b,\ t)=T_e(t),\ \frac{\partial U(0,\ t)}{\partial x}=\frac{\partial U(b,\ t)}{\partial x}=0.$$

To obtain the solution of the problem we employ the method used in [3]. Transforming (2) according to Laplace, after carrying out algebraic transformations we obtain

$$\text{a)}\ \frac{dT(p)}{dx}-\delta T(p)=\frac{\beta}{b}U(p);$$

$$\text{b)}\ T(p)=\frac{p\tau_m+\beta}{\beta}U(p)-\frac{a^2\tau_m}{\beta}\frac{d^2U(p)}{dx^2}-\frac{N(p)}{\alpha l},\tag{3}$$

where $\delta=(p\tau_l+\beta)/b$.

By means of (3a) we express the transform of the function $T_e(t)$ in terms of the transform of the function $U(t)$:

$$T_e(p)=\frac{\beta}{b}\exp(-\delta b)\int_0^b U(p)\exp(\delta x)\,dx.\tag{4}$$

The function $U(p)$ is found from the equation

$$\frac{d^3U(p)}{dx^3}+\delta\frac{d^2U(p)}{dx^2}-a_1\frac{dU(p)}{dx}-a_2U(p)=a_3P(p)+a_4\frac{dP(p)}{dx},\tag{5}$$

where

$$a_1=(p\tau_m+\beta)/a^2\tau_m,\ a_2=(p^2\tau_m\tau_l+p(\tau_m+\tau_l)\beta)/(a^2\tau_m b),$$
$$a_3=\delta a_4,\ a_4=-\beta/(\alpha l a^2\tau_m),$$

which is obtained by excluding $T(p)$ from the system (3).

Solving Eq. (5) for the boundary conditions

$$\left.\frac{\partial U(x,\ p)}{\partial x}\right|_{x=0}=\left.\frac{\partial U(x,\ p)}{\partial x}\right|_{x=b}=0,$$

taking into account the relation

$$\int_0^b U(x,\ p)\,dx=\left[-T_e(p)+\frac{\delta}{\alpha l}\int_0^b N(p)\,dx\right]\Big/W,$$

where

$$W=[p^2\tau_m\tau_l+p(\tau_m+\tau_l)\beta]/(b\beta),$$

which can be found by integrating the system of equations (3), and substituting the value of $U(p)$ thus found into the expression (4), we find

$$T_e(p)=\frac{\beta}{b}\exp(-\delta b)\left[C_{01}+\int_0^b f\exp(\delta x)\,dx\right]\left[1-\frac{\beta}{b}C_{02}\exp(-\delta b)\right]^{-1},\tag{6}$$

where f is the particular solution of Eq. (5);

$$C_{0j}=\sum_{i=1}^{3}C_{ij}[\exp((\delta+r_i)b)-1]/(\delta+r_i),\quad j=1,\ 2;$$

$$C_{ij}=\frac{d_{ij}}{d},\ d_{i1}=g_3r_{i+1}r_{i+2}[\exp(r_{i+2}b)-\exp(r_{i+1}b)]+$$
$$+[g_2-g_1\exp(r_{i+2}b)]\frac{r_{i+2}}{r_{i+1}}[\exp(r_{i+1}b)-1]+[g_1\exp(r_{i+1}b)-g_2]\times$$
$$\times\frac{r_{i+1}}{r_{i+2}}[\exp(r_{i+2}b)-1],\ d_{i2}=-\frac{r_{i+1}r_{i+2}}{W}[\exp(r_{i+2}b)-\exp(r_{i+1}b)];$$

$$d=\sum_{i=1}^{3}\frac{r_{i+1}r_{i+2}}{r_i}[\exp(r_ib)-1][\exp(r_{i+2}b)-\exp(r_{i+1}b)];$$

$$g_1=-\left.\frac{\partial f}{\partial x}\right|_{x=0},\ g_2=-\left.\frac{\partial f}{\partial x}\right|_{x=b},\ g_3=-\int_0^b f\,dx+\frac{\delta}{\alpha lW}\int_0^b N(p)\,dx;$$

the quantities r_1, r_2, and r_3 are the roots of the equation

$$y^3+\delta y^2-a_1 y-a_2=0, \tag{7}$$

with $r_i \equiv r_{i+3}$.

Thus, using (6), the sought temperature of the heat carrier at the output of the receiver $T_e(t)$, when radiation power $P(t)$ acts on it, can be found from the expression

$$T_e(t)=L^{-1}[T_e(p)],$$

where L^{-1} is the operator of the inverse Laplace transform.

Having determined the transfer function of the receiver as a ratio of power withdrawable from it to the absorbed power of radiation, transformed according to Laplace,

$$K(p)=c\gamma v T_e(p)/P(p), \tag{8}$$

by means of (6) we find

$$K(p)=\frac{c\gamma v}{P(p)}\,\frac{\beta}{b}\,\exp(-\delta b)\left[C_{01}+\int_0^b f\exp(\delta x)\,dx\right]\left[1-\frac{\beta}{b}\,C_{02}\exp(-\delta b)\right]^{-1}. \tag{9}$$

Putting $p=i\omega$ in (9), we can find the frequency (amplitude and phase) characteristics of the receiver.

We note certain general properties of the expressions obtained above. As is seen from (9), the temperature $T_e(t)$ depends on the parameters characterizing the receiver, on the variation of radiation power with time, and on the distribution of the radiation power density over the section of the beam. In accordance with the physical meaning, the transfer coefficient of the receiver at "zero" frequency in the absence of losses into the surrounding medium is 1; therefore, $K(0)\equiv 1$. In view of the fact that the temperature in the receiver cannot fall below the level $T=0$, the relation $|K(i\omega)|\le 1$ must be fulfilled; since $T_0(t)=0$, then $K(\infty)\equiv 0$.

Since the expressions (8) and (9) in the general case are fairly complex to analyze, we proceed to consider particular cases.

In the case of uniform illumination of the surface of the receiver by radiation power varying with a jump through the quantity P/A, by means of (8) and (9) we find the frequency characteristic $K(i\omega)$ and the temperature $T_e(t)$ of the receiver.

1) For $a^2=\infty$ and $\beta=\infty$:

$$\text{a) } K(i\omega)=1/[1+i\omega(\tau_m+\tau_l)],$$

$$\text{b) } T_e(t)=\frac{P}{c\gamma v}\,[1-\exp(-t/(\tau_m+\tau_l))]. \tag{10}$$

2) For $\tau_m=0$:

$$\text{a) } K(i\omega)=[1-\exp(-i\omega\tau_l)]/(i\omega\tau_l),$$

$$\text{b) } T_e(t)=\frac{P}{C_l}\,[t-h(t-\tau_l)(t-\tau_l)]. \tag{11}$$

3) For $a^2=0$:

$$\text{a) } K(i\omega)=\beta\left\{1-\exp\left[-\frac{\omega^2\tau_l\tau_m-i\omega\beta(\tau_m+\tau_l)}{\beta+i\omega\tau_m}\right]\right\}[i\omega[\beta(\tau_m+\tau_l)+i\omega\tau_m\tau_l]]^{-1},$$

$$\text{b) } T_e(t)=\frac{P}{c\gamma v(\tau_l+\tau_m)}\left\{t-\tau\left[1-\exp\left(-\frac{t}{\tau}\right)\right]-\frac{\beta}{\tau_m}\times\right.$$
$$\times\exp(-\beta)\,h(t-\tau_l)\int_0^{t-\tau_l}\left[\frac{\tau\tau_m}{\tau_l}\left[1-\exp\left\{-\frac{(t-\tau_l-\sigma)}{\tau}\right\}\right]+\right.$$
$$\left.\left.+t-\tau_l-\sigma\right]\exp\left(-\frac{\sigma\beta}{\tau_m}\right)I_0\left(2\beta\sqrt{\frac{\sigma}{\tau_m}}\right)d\sigma\right\}, \tag{12}$$

where I_0 is the modified Bessel function of the first kind of zero order;

$$\tau=\frac{C_l C_m}{alb(C_l+C_m)};\quad h(t-\tau_l)\begin{array}{ll}=0, & \text{if } t<\tau_l,\\ =1, & \text{if } t\geqslant\tau_l.\end{array}$$

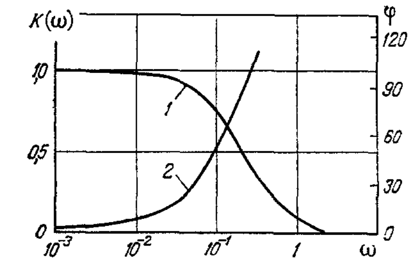

Fig. 1

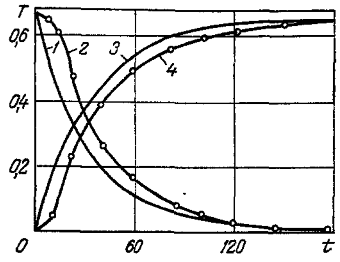

Fig. 2

Fig. 1. Amplitude–frequency (curve 1) and phase–frequency (curve 2) characteristics of HRMC: φ is the phase difference of the output and input signals, deg; ω is the cyclic frequency, rad/sec.

Fig. 2. Response of the receiver for the variation of power entering into the receiver with a jump (T, °C; t, sec). Curves 1 and 3 are calculated according to the expressions (12b); curves 2 and 4 are experimental.

4) For $b=\infty$ the expressions for $K(i\omega)$ and $T_e(t)$ coincide, respectively, with the expressions (11a) and (11b) if in them τ_l is replaced with $\tau_l + \tau_m$.

The expressions (10), as was to be expected, coincide with the corresponding expressions for receivers whose material is characterized by an infinitely large thermal diffusivity.

We note that the expressions (10) can be used in the case of finite values of b, a^2, β, while the expressions (11) and (12) can be used for values of a^2, τ_m different from zero, such that these expressions with a required accuracy differ from the initial expressions (10)-(12).

We proceed to analysis of the particular cases. It is physically clear that in the case 1 the expressions (10) also remain valid when the radiation has an arbitrary distribution of power density over the section of the beam, while the receiver has a complex form. This case takes place when we can assume that in the course of the dynamic processes the temperature in all parts of the HRMC volume is the same (volume equalization of the temperature).

Case 2 takes place when the heat capacity of the plate is fairly small. Here the thermal processes in it are established comparatively rapidly and do not exert a substantial effect on the duration of the dynamic processes in the receiver. To this case, in particular, belong receivers in which laser radiation is absorbed in the heat carrier.

In case 3, owing to the low thermal diffusivity of the material of the plate, the heat transfer in the receiver is effected only by the stream of HC exchanging with the plate.

In case 4 a local equality of temperatures is established in the volume of the receiver. In this case we can also expect that the frequency characteristic $K(i\omega)$ and the temperature at the output of the receiver $T_e(t)$ will weakly depend on the distribution of power density over the section of radiation beam and on the form of the receiver, under the condition that they insignificantly vary at distances less than b.

We carry out a comparison of the speeds of the receivers, the heat capacities $C_m + C_l$, and the velocities of volumetric flows of HC which are equal. The time of establishment of the temperature at the output of the receiver t_{es} in accordance with (10a), (11a), (12a) are as follows: In case 1) $t_{es} = 3(\tau_m + \tau_l)$; in case 2) $t_{es} = \tau_l$; in case 3) $\tau_m + \tau_l < t_{es} < (\tau_m + \tau_l) + 3\tau$ (for the cases 1 and 3 the time of establishment is defined at the level 0.97). Hence, we see that the speediest response is possessed by the receivers belonging to case 2. This is explained by the fact that in view of the smallness of τ_m, the effect of the material of which the receiver is made does not tell on the speed of response of the receiver.

In the role of an example we have presented in Fig. 1 the amplitude–frequency (curve 1) and the phase–frequency (curve 2) characteristics of HRMC with the following parameters: $a^2 = 1$ cm²/sec; b = 30 cm; l = 30 cm; m = 0.3 cm; v = 100 cm³/sec; $\alpha = 0.02$ cal/cm²·deg·sec; $C_l = 450$ cal/deg; $C_m = 153$ cal/deg; $\tau_l = 4.5$ sec; $\tau_m = 1.5$ sec; water is used in the role of the heat carrier. In Fig. 2 we have presented experimental data (curve 1, during feeding; curve 2, during removal of power) for a receiver of complicated form [1] made of copper with m = 0.15 cm, $C_m = 300$ J/deg; $C_l = 710$ J/deg; b = 300 cm; l = 100 cm. In the role of heat carrier we used water pumped through the receiver at a rate of v = 7.3 cm³/sec; curve 3 in the figure is constructed according to the expression (12b). As is seen from the figure, for the receiver $t \geq 10$ sec the agreement of the relations can be regarded as satisfactory.

Thus, the results obtained above allow us to carry out an estimate of the speed of response of HRMC from below, their dynamic properties with the basic thermophysical and geometric parameters of the receivers being taken into account.

The results obtained in the paper can be used to calculate cooling systems of gas-discharge pipes, solid-body active media, and regenerators.

NOTATION

HRMC, heat receiver with a moving heat carrier; c, specific heat capacity of liquid heat carrier; v, volumetric flow rate of heat carrier; γ, density of heat carrier; C_m, heat capacity of heat-conducting material of HRMC; C_l, heat capacity of heat carrier; α, coefficient of heat transfer at the heat-conducting material—heat carrier interface; A, absorption coefficient of HRMC; a^2, coefficient of thermal diffusivity of heat-conducting material; k, coefficient of thermal conductivity of the heat-conducting material; b, l, m, length, width, and thickness of the heat-conducting material of HRMC; β, interaction parameter; p, complex variable; T(x, t), U(x, t), functions describing the variation of temperature of the heat carrier and of temperature of the heat-conducting material along the direction of propagation of the flow with time; $T_0(t)$, $T_e(t)$, functions describing the variation of temperature, respectively, at the entry and at the exit of HRMC; N(x, t), radiation power absorbed by the receiver, referred to unit length; P(t), radiation power absorbed by the receiver; T(p), U(p), N(p), functions T(x, t), U(x, t), N(x, t) transformed according to Laplace; $T_0(p)$, $T_e(p)$, P(p) transforms of the functions $T_0(t)$, $T_e(t)$, P(t), transformed according to Laplace.

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INVESTIGATION OF THE THERMOPHYSICAL CHARACTERISTICS OF CRYOGENIC HEAT PIPES WITH A METAL-FIBER WICK

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Results are presented of an experimental investigation of the maximum heat-transmission capability and the heat-transfer intensity characteristics in the heat input and heat output of cyrogenic heat pipes.

The reliability of efficient and compact heat-transfer devices of the heat-pipe type ensure that they are widely used in practice over the whole technically available temperature range, including the cryogenic range [1-4]. An analysis of experimental and theoretical investigations [1-7] shows that the primary feature for efficient operation of cryogenic heat pipes is choice of the heat structure for the heat pipe, since the characteristic parameter $N=\sigma r/\nu$ and the thermal conductivity of cryogenic liquids are significantly lower than for low-temperature liquids (water, alcohol, or acetone). Therefore, metal-fiber wicks [8] can be regarded as one of the most promising capillary-porous structures for heat pipes in liquid nitrogen. The technology for manufacture of metal-fiber structures from monodiscrete fibers of a specific length is such that one can reduce to a minimum the thermal contact resistance between the wall and the wick and can substantially increase

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